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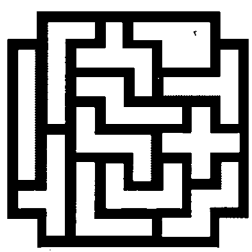
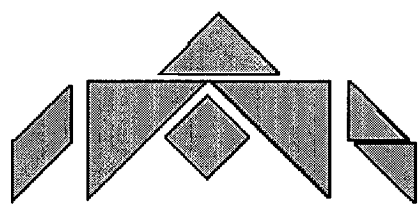
ABSTRACT

This collection of activities is designed to show how graphics display calculators can be used to explore pentominoes and tangrams. Activities include games and puzzles developing spatial relations. Teaching notes, solutions, and calculator instructions are included as are blackline masters. (MM)

Pentominoes & Tangrams
High School Explorations





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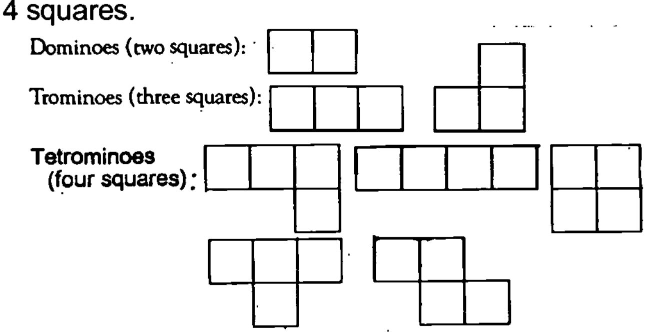
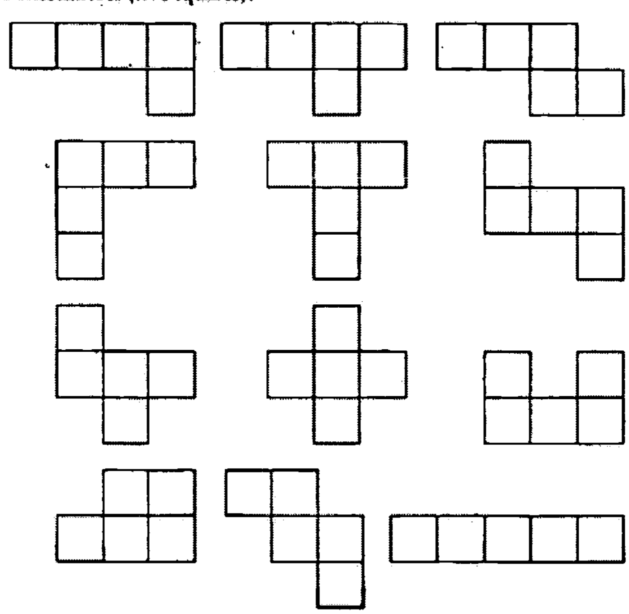
Activity 1 - Making a set of Pentominoes

Each Student will receive five one-inch squares. Using all five of the squares, students will make as many different figures as possible. A domino is constructed from 2 squares juxtaposed along the sides, a tromino from 3 squares, and a tetromino from 4 squares.



A pentomino is constructed from 5 such squares, each having at least one of its sides along that of another. Students then sketch these on one-inch grid paper and cut out. Their set of 12 pieces, shown below, can be stored in an envelope for future activities and investigations. Alternatively, the teacher can give the students a template of the shapes as seen in Activity 2 or Activity 4 to cut out and store.

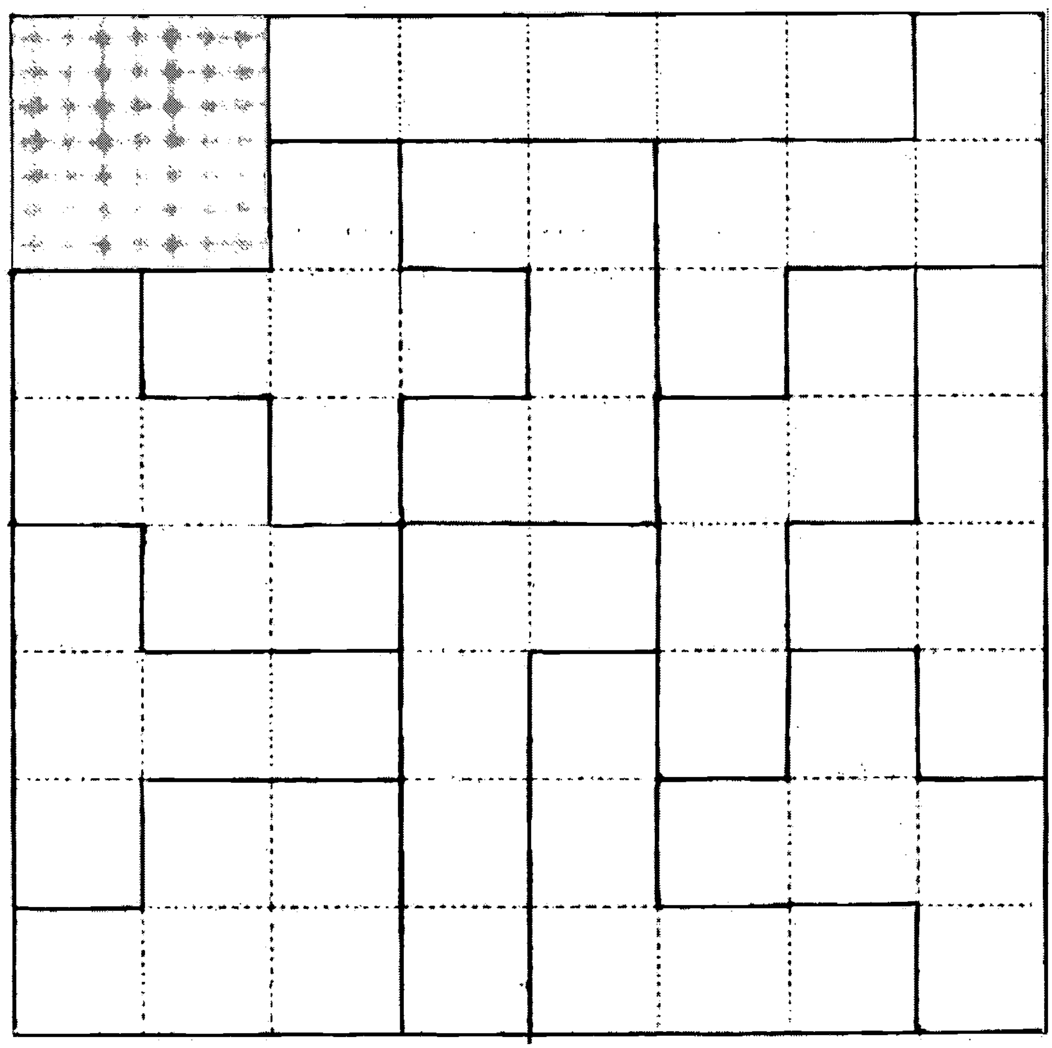
Pentominoes (five squares):



Activity 2 - The Game of P-E-N-T-O-M-I-N-O-E-S

Students will use one set of pentomino pieces and an eight by eight playing grid. By playing this game the students will be learning about the shapes and about spatial relationships. (See next page)

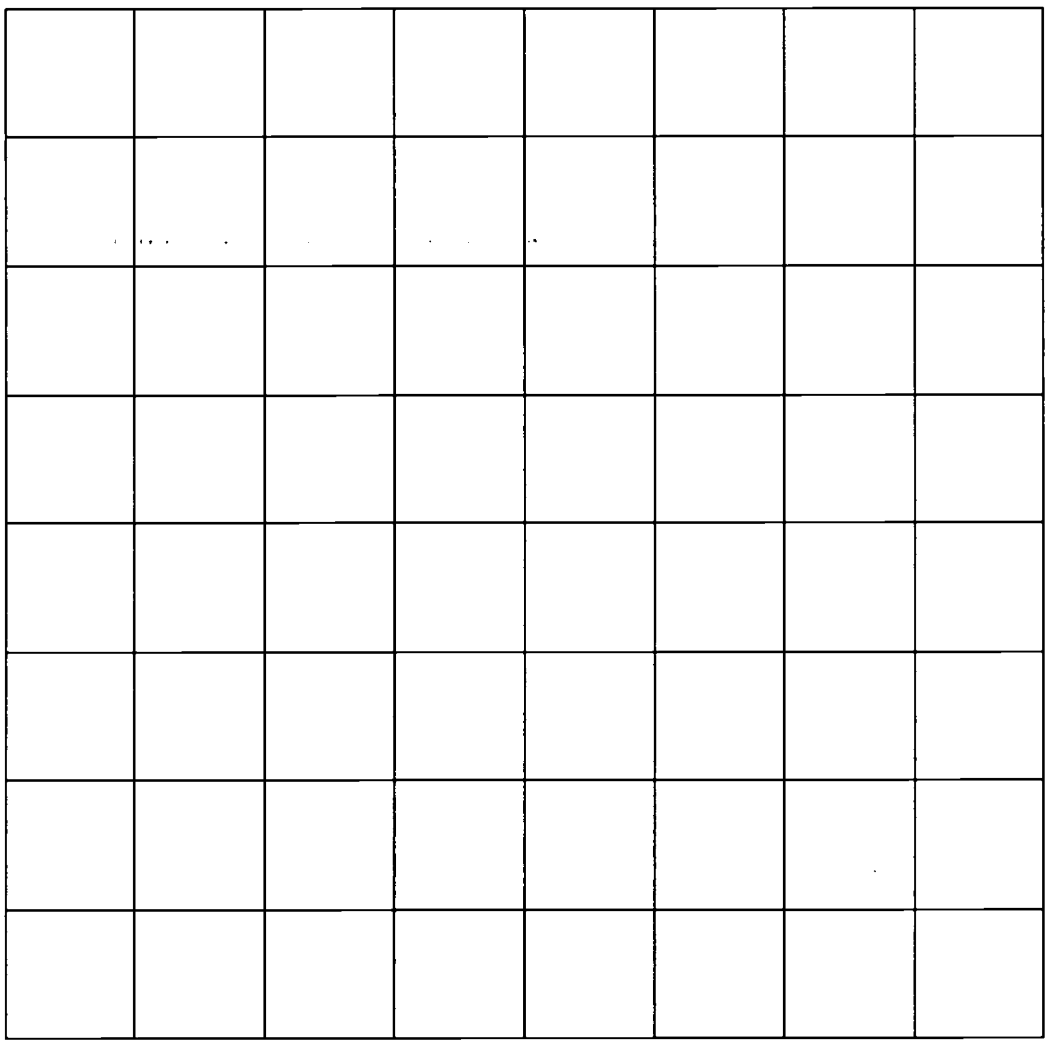
Cut Along the solid lines to form the twelve Pentomino pieces.



The Game of P-E-N-T-O-M-I-N-O-E-S

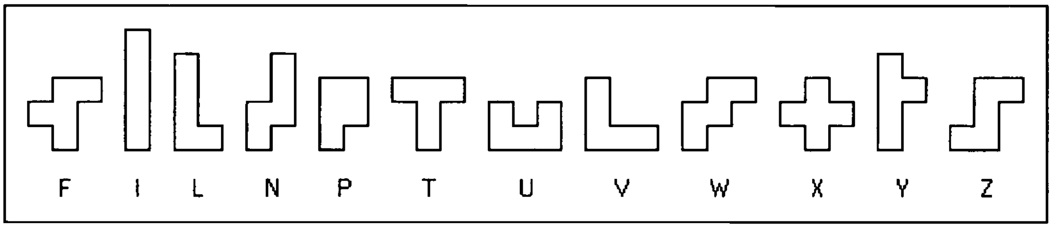
Directions:

Players take turns choosing a piece and placing it on the board. The pieces must not overlap or extend beyond the boundaries of the board, but they do not have to be adjacent. Tiles may be flipped. The object of the game is to be the last player to place a pentomino piece on the 8 X 8 grid. Two or three players can play the game. Students can come up with their own playing rules. Tournaments can also be scheduled.



Activity 3 - Investigating Properties of the Pieces

In this activity students will be filling in the table below. They can use their constructed pentomino pieces to help fill it in:



TILE	AREA	PERIMETER	CLASSIFY BY SIDES	SYMMETRIES REFLECTIONS	SYMMETRIES ROTATIONS	OPEN TOP BOX (Y/N)
F						
I						
L						
N						
P						
T						
U						
V						
W						
X						
Y						
Z						

EXTENSIONS:
New columns can be added to investigate other properties as needed.

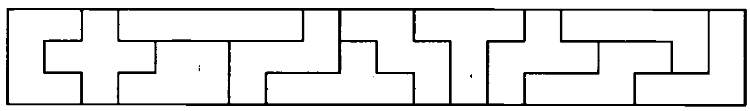
Activity 4: Filling the Space

In this activity students will take their pentomino tiles and construct given shapes.

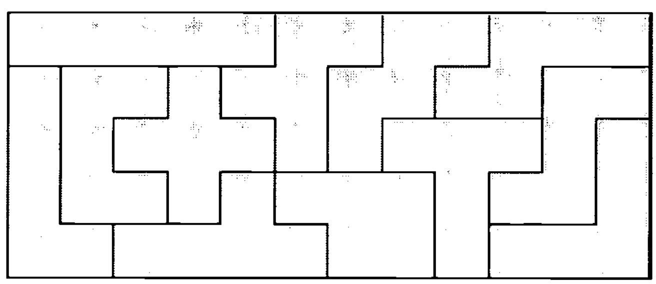
For instance:

1. Place the pieces to form a 3x20 rectangle.
2. Place the pieces to form a 5X12 rectangle.
3. Place the pieces to form a 6X10 rectangle.
4. Place the pieces to create your own design, trace the outline, and then challenge a classmate.
5. Choose 5 pieces and use them to form a square.

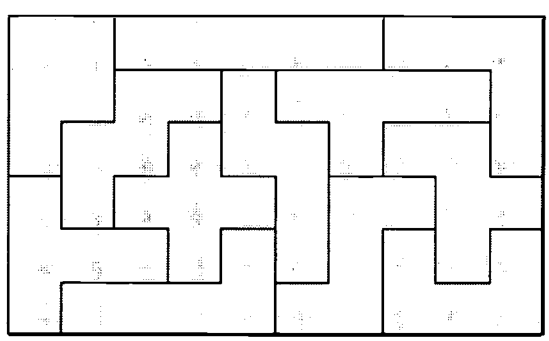
3X20 grid:



5X12 grid:



6X10 grid:



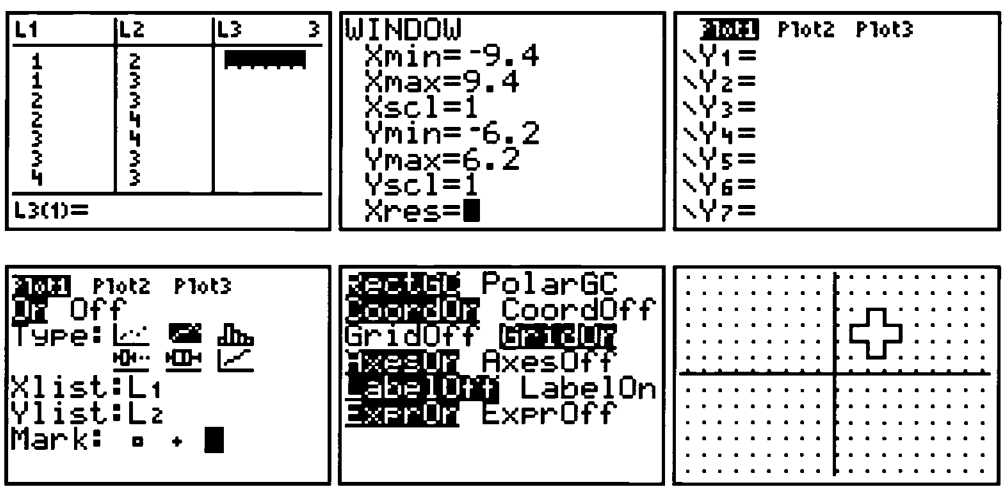
Activity 5: Graphing a Pentomino

The object of this activity is to graph one of the pentomino pieces in the first quadrant on the graphing calculator. Figure out what coordinates should be placed in L1 and L2 so that using the line graph in StatPlot for L1 and L2 you will produce a sketch of the pentomino piece.

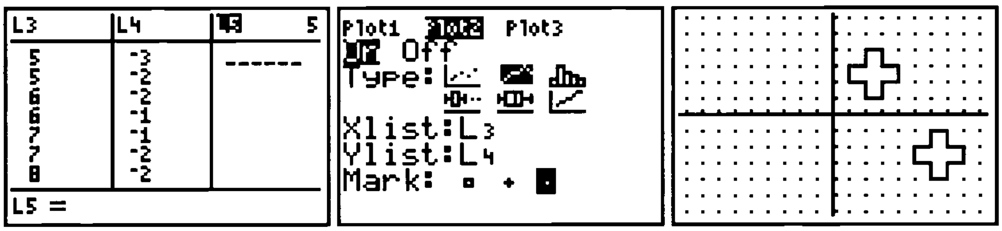
To create the line plot of a pentomino piece, press STAT, EDIT, Enter. Clear out lists L1 and L2 and enter the x coordinates of the points in L1 and the y coordinates in L2. To create a closed figure reenter the first point again as the last point on the list. The following table contains the points necessary for the "X" piece.

x	1	1	2	2	3	3	4	4	3	3	2	2	1
y	2	3	3	4	4	3	3	2	2	1	1	2	2

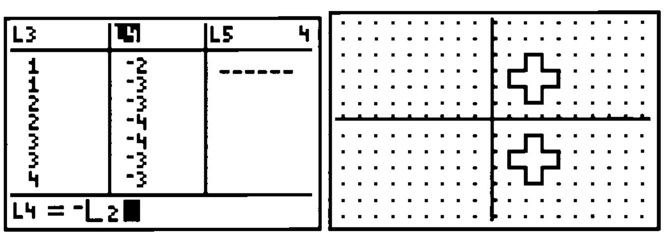
Set a friendly window of [-9.4,9.4,1, -6.2,6.2,1]. Clear any functions in Y=. Press 2nd STATPLOT and select Plot1. Turn the plot on, choose the second type graph that is a line graph, select L1 as Xlist and L2 as Ylist, and finally select the Mark. Before graphing it might be more convenient if a grid is displayed. To accomplish this, press 2nd FORMAT, scroll to GridOn and select it. Now press graph.



Students can now translate their piece. In STAT, EDIT scroll to L3 and highlight it with the cursor. Enter L1 + 4 and press enter. Scroll to L4, highlight it and enter L2 – 5 and press enter. Press 2[nd] STATPLOT and select plot2. Turn the plot on, choose the second type graph that is a line graph, select L3 as Xlist and L4 as Ylist, and finally select the Mark. Press graph. The original piece and the translated piece will both be graphed. In the above example the “X” was shifted right 4 units and shifted down 5 units.



Now, to reflect their piece in the x-axis, scroll to L3 in the Stat, Edit. Highlight it with the cursor and enter L1 and press enter. Next Scroll to L4, highlight it, enter –L2 and press enter. When graphed, Plot1 shows the original piece and Plot2 shows the reflected piece.



Challenge:

- Sketch the translation of the piece to the left 3 units, down 2 unit, and then reflected in the x-axis.
- Create your own combinations of transformations.

Activity 6 – Making a set of TANGRAMS

Each student will receive a square sheet of paper. The teacher will give the following directions to the students to construct their Tangram pieces.

1) Take the square and fold one corner over to its opposite corner. Cut on the diagonal fold.

2) Take one of these triangles and set the other aside. Take the triangle and again, fold one so the right angle is bisected. Cut on the angle bisector fold.

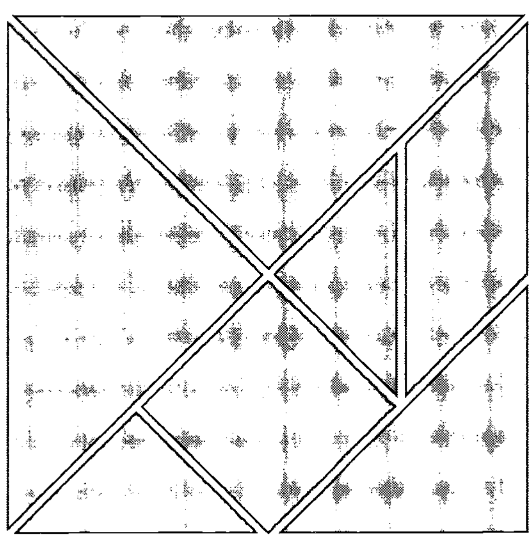
3) Set these smaller triangles aside and take the remaining larger triangle. Find the midpoints of the two congruent legs. Fold on the segment joining these midpoints. Cut on the midpoint connector fold.

4) Place the small triangle aside and take the isosceles trapezoid and fold it on its line of symmetry. Cut along this line.

5) Take one of the smaller trapezoids and find the midpoint of the longer base. Fold from that midpoint to the vertex of the obtuse angle. Cut along the fold and set aside the square and small right triangle.

6) Now take the other trapezoid and fold it so that the vertex of the obtuse angle coincides with the opposite right angle. Cut on this fold to form a parallelogram and another right triangle.

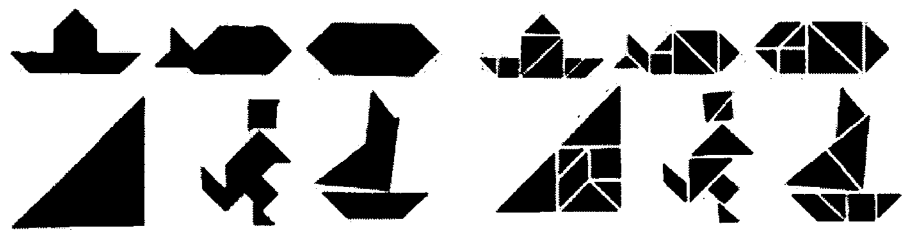
We now have the seven shapes of the Tangram. Below is a template that can be used to form the seven Tangram pieces.



Activity 7- Putting it all back Together

- Challenge your students to take the seven Tangram pieces and put them back together to form the larger square.

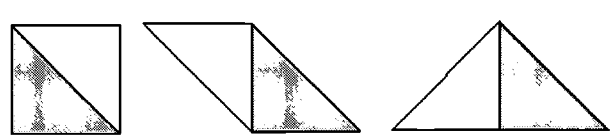
- Challenge your students to take all seven Tangram pieces and create one of the pictures below. Answers at bottom of page.



- Challenge your students to take the seven Tangram pieces and create their own designs. Once they have created a design have the students break down into pairs and sit back to back. Each student in turn describes, using the proper vocabulary, their design to their partner who is recreating the design with their set of Tangram pieces. The partners can compare pictures and results.

- Challenge your students to create squares using different numbers of Tangram pieces. For example, to form a square with one Tangram piece, students should identify the square piece. To form a square with two Tangram pieces, students should use the two small triangles or the two large triangles. Students should continue to try to form squares with 3 pieces, 4 pieces, 5 pieces, 6 pieces and all 7 pieces. Do you notice any patterns? One of these is impossible. Which one is it, and why isn't it possible?

- In each of the three pictures above, the gray triangle is in the same position.



Describe how to move the white triangle (rotations, translations, and reflections) to get from the square to the parallelogram, from the square to the triangle, and from the parallelogram to the triangle.

Activity 8 – Areas

Below are the seven Tangram Shapes. There are two large triangles, one medium triangle, two small triangles, a square and a parallelogram.

Fill-in the following table given the area of one of the pieces:

	Large Triangle	Medium Triangle	Small Triangle	Square	Parallelogram
Area	1				
Area		1			
Area			1		
Area				1	
Area					1

Dissecting a large square formed the seven Tangram pieces. If the area of the large square is one, what is the area of each one of the pieces?

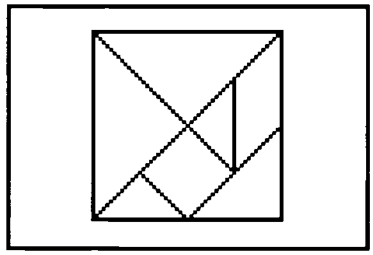
Extensions:
What are the angle measures of each piece?

If a side of the small square is 1, find the perimeter of each of the pieces, as well as the large square we started with.

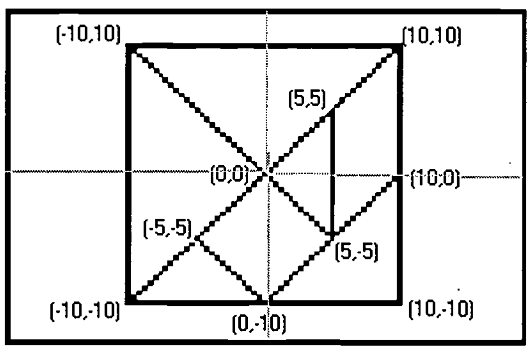
If the area of the large original square is 4, what are the areas and perimeters of each of the seven Tangram shapes?

Discuss the symmetries, congruencies and similarities of the seven Tangram shapes.

Activity 9 - Graphing the Tangram Puzzle



The object of this activity is to get the students to get a picture of the Tangram puzzle on the screen. One way of doing this is similar to *Activity 5- Graphing the Pentomino*. Students will need to find the coordinates of the vertices of the Tangram figure. They can do this by plotting the Tangram on graph paper. I chose the following scale:



The coordinates are then placed in L1 and L2 so that a line graph can be formed. Students will need to repeat coordinates so the figure is connected properly. One such list, containing 14 elements, is given below. Challenge the students to come up with as short a list as possible that will still accomplish the task of generating the Tangram puzzle.

List1	10	-10	-10	10	10	-10	0	10	5	5	-5	0	5	-10
List2	10	10	-10	-10	10	-10	-10	0	-5	5	-5	-10	-5	10

As an alternative, students can enter piecewise functions in Y1, Y2, etc. and use the Line command in 2nd Draw to generate the picture of the Tangram puzzle. The following figures show how to set it up.

After setting up the window, turning the axes off, and plotting the 6 piecewise functions it is necessary to use the Line command to draw in the three vertical lines. Press 2nd Draw, select Line and enter the coordinates of the points to be connected. Press enter and the line will be graphed. Press 2nd Quit to enter the second line and then repeat to get the last line. A TI 83 program, **TANGRAM**, is given below that will sketch the Tangram puzzle.

TANGRAM

```
:ClrDraw
:Func
:Simul
:AxesOff
:ú18üXmin:18üXmax
:ú12üYmin:12üYmax
:"X/((Xùú10)*(X÷10))"üY•
:"10/((Xùú10)*(X÷10))"üY,
:"ú10/((Xùú10)*(X÷10))"üYƒ
:"(X-10)/((Xù0)*(X÷10))"üY„
:"úX/((Xùú10)*(X÷5))"üY…
:"(úX-10)/((Xùú5)*(X÷0))"üY†
:Line(5,ú5,5,5)
:Line(ú10,ú10,ú10,10)
:Line(10,ú10,10,10)
```

PENTOMINO FACTOIDS

The "invention" of Pentominoes is generally credited to Solomon W. Golomb in 1953 at a talk he gave to the Harvard Mathematics Club. He coined the name pentomino.

Alexey Pajnitov's inspiration for Tetris was the traditional game of Pentominoes.

There are 12 different pieces in Pentominoes, 35 pieces in Hexominoes, 108 pieces in Heptaminoes, 369 pieces in Octominoes, 1285 pieces in Nonominoes and 4655 in Decominoes.

To fit the 12 Pentominoes into a 6 X 10 rectangle, there are 2,339 solutions. To fit the same shapes into a 3 X 20 rectangle, there are only 2 solutions.

Solid Pentominoes are the three-dimensional version of Pentominoes. The most common objective is to arrange them into a 3 X 4 X 5 solid. There are 3,940 ways of doing it. A 2 X 5 X 6 or a 2 X 3 X 10 are also possible solids to make.

Resources – Links:

http://www.snaffles.demon.co.uk/pentanomes/5x12_singles.html
http://www.theory.csc.uvic.ca/~cos/inf/misc/PentInfo.html
http://forum.swarthmore.edu/pom/project2.95.html
http://userpage.chemie.fu-berlin.de/~lorente/lien.html
http://godel.hws.edu/java/pent1.html

Tangram Factoids

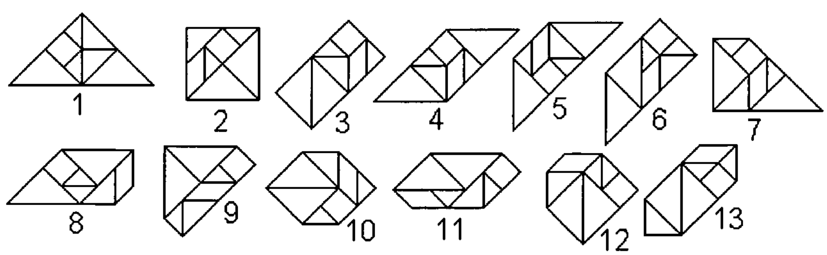
Tangrams is a very old game originating in China. There are hundreds of shapes that can be made from all seven pieces. The invention of the Tangram puzzle is unrecorded in history. The earliest known Chinese book is dated 1813 but the puzzle was very old by then. 'Tan' means 'pieces', and 'grams' means 'pictures'.

A legend: Tan, the son of a Chinese nobleman, received a square ceramic tile that he valued more than anything else he owned. Because the tile was very beautiful and Tan was very proud of it, he showed the tile to everyone he met. One day, while doing this, Tan dropped the tile. It broke into seven pieces. The legend tells us that Tan was very sad and spent the rest of his life trying to put the pieces together. He was able to make a triangle, a house, a cat, a bird, and many other interesting shapes; but he could never make the square shape of the original tile.

Tangram is an ancient Chinese game that is also known as "the wisdom puzzle."

The only piece that needs to be turned over is the parallelogram.

These shapes are the only thirteen convex polygons that can be formed with a set of Tangrams.



Resources – Links:

http://forum.swarthmore.edu/trscavo/tangrams.html
http://www.ex.ac.uk/cimt/res2/trolxk.pdf
http://www.enchantedmind.com/tangram/tangram.htm

Time Travel and Other Mathematical Bewilderments by Martin Gardner
Grandfather Tang's Story: A Tale Told with Tangrams by Ann Tompert

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